Multi-Layer Precoding for Full-Dimensional Massive MIMO Systems

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Abstract—Full-dimensional massive multiple-input multiple-output (MIMO) systems boost sum spectral efficiency by offering orders of magnitude increase in multiplexing gains. In time division duplexing systems, however, the reuse of the uplink training pilots among cells results in channel estimation errors, which lead to downlink inter-cell interference, especially for cell-edge users, even with large numbers of antennas. Handling this interference with conventional network MIMO techniques is challenging due to the high channel dimensionality. Further, large antenna precoding and combining implementation is associated with high hardware complexity. In this paper, we propose multi-layer precoding to enable efficient and low complexity full-dimensional MIMO operation. Multi-layer precoding (i) leverages the directional characteristics of large-scale MIMO channels to manage inter-cell interference with low channel requirements, and (ii) allows for an efficient implementation using low-complexity hybrid analog/digital architectures. We present and evaluate a specific multi-layer precoding design for full-dimensional MIMO systems. Simulation results show the potential gains of multi-layer precoding compared with traditional pilot-contaminated massive MIMO setups despite the low channel knowledge requirements and the low-complexity implementation.

I. INTRODUCTION

Full-dimensional massive MIMO promises significant spectral efficiency gains for cellular systems. Scaling up the number of antennas, however, faces a number of challenges that prevent the corresponding scaling of the gains. The large-dimensional channels have high feedback overhead in frequency division duplexing (FDD) systems. To overcome that, channel reciprocity in time division duplexing (TDD) systems was leveraged. Reusing the uplink training pilots among cells, however, causes channel estimation errors which in turn lead to downlink inter-cell interference, especially for cell-edge users. Handling inter-cell interference using traditional network MIMO techniques has high coordination overhead. Another challenge with the large number of antennas lies in the hardware implementation. Traditional MIMO precoding techniques generally assumes that the processing is done in the baseband. This, however, assumes the dedication of a separate RF chain per antenna, which is difficult with large numbers of antennas. Therefore, developing precoding schemes that can overcome these challenges is of great interest.

Motivated by the hardware limitations on the RF chains in millimeter wave large antenna systems, [1], [2] proposed to divide the precoding processing between the analog and digital domains [3], known as hybrid analog/digital precoding. In [1] and [2], low-complexity hybrid precoding algorithms were developed for single-user and multi-user systems, exploiting the sparse nature of millimeter wave channels. These algorithms, however, did not account for out-of-cell interference. In another research direction, [4] proposed the joint spatial division and multiplexing scheme, with the aim of reducing the channel feedback overhead in FDD massive MIMO systems. In this scheme, the base station (BS) divides the mobile stations (MS’s) into groups of approximately similar covariance eigenspaces, and designs a pre-beamforming matrix based on the large channel statistics. The interference between the groups of users is then managed using another precoding matrix given the effective reduced-dimension channels. The work in [4], however, did not consider out-of-cell interference. In [5], the directional characteristics of large-dimensional channels were leveraged to improve the uplink channel training in TDD systems, with the knowledge of the interference covariance matrix, but did not account for hardware constraints.

In this paper, we introduce a generalization of hybrid precoding called multi-layer precoding. The idea is to decompose the precoder into a product of different precoding matrices, each designed based on a different performance objective. The approach is applied to full dimension massive MIMO systems to allow for inter-cell interference management using large-scale channel statistics, and for efficient implementation using hybrid analog/digital architectures with a relatively small number of RF chains. We propose and evaluate a specific multi-layer precoding design that leverages the low-rank property of the elevation covariance matrix and the directional characteristic of large-dimensional channels to realize a low-complexity solution. The proposed technique is also evaluated by simulations in a cellular setup where a noticeable coverage gain was shown compared with pilot-contaminated traditional massive MIMO solutions.

We use the following notation: A is a matrix, a is a vector, a is a scalar, and A is a set. A*, A⁻¹, R(A) are the Hermitian, inverse, and range of A, respectively. A ⊗ B is the Kronecker product of A and B, and A o B is their Khatri-Rao product.

II. SYSTEM MODEL

Consider a cellular system model consisting of L cells with one BS and K MS’s in each cell, as shown in Fig. 1. Each
BS is equipped with a two-dimensional (2D) antenna array of \( N_V \) (vertical antennas) \( \times N_H \) (horizontal antennas), and each MS has a single antenna. We assume that all BS’s and MS’s are synchronized and operate a TDD protocol with universal frequency reuse. In the downlink, each BS \( \ell, \ell = 1, 2, \ldots, L \), applies a \( N_V N_H \times K \) precoder \( \mathbf{F}_\ell \) to transmit a symbol for each user, with a power constraint \( \| \mathbf{F}_\ell \|_2^2 = 1 \), \( k = 1, 2, \ldots, K \). Uplink and downlink channels are assumed to be reciprocal. If \( \mathbf{h}_{\ell ck} \) denotes the \( N_V N_H \times 1 \) uplink channel from user \( k \) in cell \( c \) to BS \( \ell \), then the received signal by this user in the downlink can be written as

\[
y_{\ell ck} = \sum_{\ell=1}^{L} \mathbf{h}_{\ell ck}^* \mathbf{F}_\ell \mathbf{s}_\ell + n_{\ell ck},
\]

where \( \mathbf{s}_\ell \) is the \( K \times 1 \) vector of transmitted symbols from BS \( \ell \), such that \( \mathbb{E} [\mathbf{s}_\ell \mathbf{s}_\ell^*] = \frac{P}{K} \mathbf{I} \), with \( P \) representing the average total transmitted power, and \( n_{\ell ck} \sim \mathcal{N}(0, \sigma^2) \) is the Gaussian noise at user \( k \) in cell \( c \).

Given the 2D antenna arrays deployed at the BS’s, the channels from the BS’s to each user have a 3D structure. Extensive efforts are currently given to 3D channel measurements, modeling, and standardization \([6],[7]\). One candidate is the Kronecker product correlation model, which was shown to provide a very good approximation to 3D covariance matrices \([8]\). In this model, the covariance of the 3D channel \( \mathbf{h}_{\ell ck} \), which is defined as \( \mathbf{R}_{\ell ck} = \mathbb{E} [\mathbf{h}_{\ell ck} \mathbf{h}_{\ell ck}^*] \), is approximated by

\[
\mathbf{R}_{\ell ck} = \mathbf{R}_{\ell ck}^A \otimes \mathbf{R}_{\ell ck}^E,
\]

(2)

where \( \mathbf{R}_{\ell ck}^A \) and \( \mathbf{R}_{\ell ck}^E \) represent the covariance matrices in the azimuth and elevation directions, respectively. If \( \mathbf{R}_{\ell ck}^A = \mathbf{U}_{\ell ck}^A \Lambda_{\ell ck}^A \mathbf{U}_{\ell ck}^A^* \) and \( \mathbf{R}_{\ell ck}^E = \mathbf{U}_{\ell ck}^E \Lambda_{\ell ck}^E \mathbf{U}_{\ell ck}^E^* \) are the eigenvalue decompositions of \( \mathbf{R}_{\ell ck}^A \) and \( \mathbf{R}_{\ell ck}^E \), then using Karhunen-Loeve representation, the channel \( \mathbf{h}_{\ell ck} \) can be expressed as \([4]\)

\[
\mathbf{h}_{\ell ck} = \left[ \mathbf{U}_{\ell ck}^A \Lambda_{\ell ck}^A \left( \frac{1}{2} \right) \otimes \mathbf{U}_{\ell ck}^E \Lambda_{\ell ck}^E \left( \frac{1}{2} \right) \right] \mathbf{w}_{\ell ck},
\]

(3)

where \( \mathbf{w}_{\ell ck} \sim \mathcal{N}(0, \mathbf{I}) \) is a rank \( (\mathbf{R}_{\ell ck}^A) \) \( \times 1 \) vector, with rank(\( \mathbf{A} \)) representing the rank of the matrix \( \mathbf{A} \). As the elevation direction will likely experience less scattering \([7],[9]\), the elevation covariance matrix, \( \mathbf{R}_{\ell ck}^E \), may have low rank. For the sake of tractability, we consider in this work the case when \( \mathbf{R}_{\ell ck}^E \) is a rank-1 matrix, i.e., \( \mathbf{R}_{\ell ck}^E = \lambda_{\ell ck} \mathbf{u}_{\ell ck} \mathbf{u}_{\ell ck}^* \).

In this case, the channel vector \( \mathbf{h}_{\ell ck} \) can be written as

\[
\mathbf{h}_{\ell ck} = \mathbf{h}_{\ell ck}^A \otimes \lambda_{\ell ck} \mathbf{u}_{\ell ck},
\]

(4)

with \( \mathbf{h}_{\ell ck}^A = \mathbf{U}_{\ell ck}^A \lambda_{\ell ck}^A \frac{1}{2} \mathbf{w}_{\ell ck} \). To simplify the notations, the superscript \( E \) is dropped from \( \lambda_{\ell ck} \) and \( \mathbf{u}_{\ell ck} \).

III. MULTI-LAYER PRECODING: THE GENERAL CONCEPT

In this section, we briefly introduce the motivation and general concept of multi-layer precoding. Given the system model in Section II, the signal-to-interference-plus-noise ratio (SINR) at user \( k \) in cell \( c \) is

\[
\text{SINR}_{ck} = \frac{P}{\sum_{m \neq k} \| \mathbf{h}_{\ell ck}^m \mathbf{F}_\ell \|_2^2 + \frac{P}{K} \| \mathbf{h}_{\ell ck}^k \mathbf{U}_{\ell ck} \|_2^2 + \sigma^2},
\]

(5)

where the terms \( \| \mathbf{h}_{\ell ck}^m \mathbf{F}_\ell \|_2^2 \), \( \sum_{m \neq k} \| \mathbf{h}_{\ell ck}^m \mathbf{F}_\ell \|_2^2 \), and \( \sum_{\ell \neq c} \| \mathbf{h}_{\ell ck}^c \mathbf{U}_{\ell ck} \|_2^2 \) are the desired signal power, intra-cell multi-user interference, and inter-cell interference, respectively. Our objective is to design the precoding matrices, \( \mathbf{F}_\ell, \ell = 1, 2, \ldots, L \), such that (i) they manage the inter-cell interference with low requirements on the channel knowledge, and (ii) they can be implemented using low-complexity hybrid analog/digital architectures \([2]\), i.e., with a small number of RF chains. Next, we present the main idea of multi-layer precoding, a potential solution to achieve these objectives.

Inspired by the prior work on multi-user hybrid precoding \([2]\) and joint spatial division multiplexing \([4]\), and leveraging the directional characteristics of large-scale MIMO channels \([5]\), we propose to design the precoding matrix \( \mathbf{F}_c \) as a product of a number of precoding matrices (layers). In this paper, we will consider a 3-layer precoding matrix

\[
\mathbf{F}_c = \mathbf{F}_c^{(1)} \mathbf{F}_c^{(2)} \mathbf{F}_c^{(3)},
\]

(6)

where each layer is designed to achieve only one precoding objective, e.g., maximizing desired signal power, minimizing inter-cell interference, or minimizing multi-user interference. Further, these precoding objectives are distributed over the precoding layers such that \( \mathbf{F}_c^{(1)} \) requires slower time-varying channel state information compared with \( \mathbf{F}_c^{(2)} \), which in turn requires slower channel state information compared with \( \mathbf{F}_c^{(3)} \). Finally, each precoding layer is designed based on the effective channel that includes the effect of the prior layers.

In the next sections, we will present a specific multi-layer precoding design for full-dimensional massive MIMO systems, and show how it enables leveraging the large-scale MIMO channel characteristics to manage inter-cell interference with limited channel knowledge. We will also show how the multiplicative structure of multi-layer precoding allows for efficient implementations using hybrid analog/digital architectures.

IV. PROPOSED MULTI-LAYER PRECODING SCHEME

To leverage the low-rank property of the elevation covariance matrix explained in Section II, we will focus on applying the idea of multi-layer precoding in the elevation direction. Given the Kronecker structure of the channel model in (4), we propose to design the precoding matrix \( \mathbf{F}_c \) as

\[
\mathbf{F}_c = \mathbf{F}_c^A \mathbf{F}_c^E,
\]

(7)

\[
= \mathbf{F}_c^A \mathbf{F}_c^{E(1)} \mathbf{F}_c^{E(2)} \mathbf{F}_c^{E(3)},
\]

(8)
where $F^A_c$ is an $N_H \times K$ azimuth precoding matrix designed based on the azimuth channels, and $F^E_c = F^{E(1)}_c F^{E(2)}_c F^{E(3)}_c$ is an $N_V \times K$ elevation multi-layer precoding matrix designed based on the elevation channels. Given the Khari-Rao structure of the proposed precoders, the Kronecker structure of the channel model in (4), and noting that $(A \otimes B)(C \circ D) = (AC \circ BD)$, the received signal by user $k$ in cell $c$ in (1) can be written as

$$y_{ck} = \left( h_{cck}^A F^A_c \circ \lambda_{cck}^2 U_{ck}^* F^{E(1)}_c F^{E(2)}_c F^{E(3)}_c \right) s_c + \sum_{\ell \neq c} \left( h_{cck}^A \odot \lambda_{cck}^2 u_{\ell ck}^* F^{E(1)}_\ell F^{E(2)}_\ell F^{E(3)}_\ell \right) s_\ell + n_{ck}. \tag{9}$$

### A. First Layer: Inter-Cell Interference Management

We will design the first precoding layer $F^{E(1)}_c$ to avoid the inter-cell interference, i.e., the second term of (9). Given the received signal in (9), avoiding the inter-cell interference is satisfied if $F^{E(1)}_c$ is designed such that $u_{\ell ck}^* F^{E(1)}_c = 0, \forall k, \forall \ell \neq c$. Note that when all $F^{E(1)}_\ell, \ell = 1, 2, ..., L$, are designed to satisfy this criteria, $y_{ck}$ in (9) will be free of inter-cell interference. Consequently, $F^{E(1)}_c$ needs to be designed to be in the null-space of the elevation covariance matrices of all the channels connecting BS $c$ and the users of the other cells, i.e., to be in $\mathcal{N} \left( \sum_{\ell \neq c} \sum_{k \in K_\ell} R^E_{\ell ck} \right)$ where $K_\ell$ represents the subset of $K$ scheduled users in cell $\ell$.

Thanks to the directional structure of large-scale MIMO channels, we note that with a large number of vertical antennas, $N_V$, the null-space $\mathcal{N} \left( \sum_{\ell \neq c} \sum_{k \in K_\ell} R^E_{\ell ck} \right)$ will have a large overlap for different scheduled users $K_\ell$. This means that designing $F^{E(1)}_c$ based on the average null-space over different scheduled users may be enough. Leveraging this intuition relaxes the required channel knowledge to design the first precoding layer. Hence, we define the average interference covariance matrix for BS $c$ as

$$R^I_c = \sum_{\ell \neq c} E_{K_\ell} \left[ \sum_{k \in K_\ell} \lambda_{\ell ck} u_{\ell ck} u_{\ell ck}^H \right]. \tag{10}$$

If $U^N_c$ represents the eigenvectors of $R^I_c$ that correspond to zero eigenvalues, then we design the first precoding layer $F^{E(1)}_c$ to be in the null-space of the average interference covariance matrix by setting

$$F^{E(1)}_c = U^N_c. \tag{11}$$

Given the first layer precoder design, the received signal at user $k$ in cell $c$ in (9) becomes

$$y_{ck} \approx \left( h_{cck}^A F^A_c \circ \lambda_{cck}^2 U_{ck}^* U^N_c F^{E(2)}_c F^{E(3)}_c \right) s_c + n_{ck}, \tag{12}$$

where the approximation sign is due to the expectation in (10). Note also that this approximation is especially good with large numbers of vertical antennas $N_V$ as will be illustrated in the simulations of Section VII.

### B. Second Layer: Desired Signal Beamforming

The second precoding layer $F^{E(2)}_c$ is designed to maximize the desired signal power under the effective channels, i.e., including the effect of the first precoding layer. If we define the effective elevation eigenvector matrix for cell $c$ as $U_c = U^N_c [u_{c1}, u_{c2}, ..., u_{cK}]$, then we design the second precoding layer $F^{E(2)}_c$ as a conjugate beamforming matrix, i.e., we set $F^{E(2)}_c = U_c$.

Note that designing the second precoding layer requires only the knowledge of the effective elevation eigenvector matrix $U_c$, which depends on large-scale channel statistics. Further, during the uplink training of the matrix $U_c$, the first precoding layer works as a spatial filter for the other cell interference. Hence, this reduces (and ideally eliminates) the estimation error due to pilot reuse among the cells, and consequently leads to a pilot decontamination effect [5].

Given the second layer precoder design, the received signal by user $k$ in cell $c$ will be

$$y_{ck} \approx \left( h_{cck}^A F^A_c \circ \lambda_{cck}^2 U_{ck}^* U^N_c U_c F^{E(3)}_c \right) s_c + n_{ck}. \tag{13}$$

### C. Third Layer: Multi-User Interference Management

The third precoding layer $F^{E(3)}_c$ is designed to manage the multi-user interference based on the effective channel, i.e., including the effect of the first and second precoding layers. If we define the $K \times K$ effective elevation channel matrix as $H^{E}_{c,\text{eff}} = D^2_{\ell c} U_c^H U_c$, with $D = \text{diag}(\lambda_{c1}, \lambda_{c2}, ..., \lambda_{cK})$, we then design the precoder $F^{E(3)}_c$ as a zero-forcing matrix, i.e., we set $F^{E(3)}_c = H_{c,\text{eff}}^{-1} \left( H_{c,\text{eff}}^H H_{c,\text{eff}} \right)^{-1} Y_c$, where $Y_c$ is a diagonal normalization matrix that ensures satisfying the precoding power constraint.

Finally, as the multi-user interference is already handled using the precoder $F^{E(3)}_c$, we design $F^A_c$ as a conjugate beamforming matrix based on the azimuth channels, i.e., we set $F^A_c = \frac{h_{cck}^A}{\|h_{cck}^A\|}$, $k = 1, 2, ..., K$. It may, however, be interesting for future work to investigate different joint designs of $F^{E(3)}_c$ and $F^A_c$.

Given the design of the precoding matrices $F^{E(3)}_c$ and $F^A_c$, the received signal by user $k$ in cell $c$ is written as

$$y_{ck} \approx \|h_{cck}^A\| [Y_c]_{k,k} [s_c]_k + n_{ck}. \tag{14}$$

Note that while we focused in this paper on TDD systems, the proposed multi-layer precoding design requires only large-scale channel statistics to design the first and second precoding layers, and a very small channel matrix (compared with the original channel dimensions) to design the third precoding layer. This makes multi-layer precoding schemes attractive for FDD systems as well.

### V. Achievable Rates

In this section, we characterize a lower bound on the achievable rates using the proposed multi-layer precoding scheme. For tractability, we adopt the following decoupling assumption between the signal and interference elevations eigenspaces.
Assumption 1: The eigenvectors of the signal and interference elevation channel covariance matrices of each cell $c$ satisfy

- $u_{ckk} \in \mathcal{R}(U_{c}^{N_{c}})$, $\forall k$
- $u_{ckk} \in \mathcal{R}(U_{c}^{N_{c}})^{\perp}$, $\forall k, \forall k \neq c$

Note that this decoupling assumption is a good assumption at large numbers of vertical antennas $N_{V}$.

Proposition 2: Consider the system model in Section II, and Assumption 1, the achievable rate of user $k$ in cell $c$ when the multi-layer precoding scheme in Section IV is used to construct the BS downlink precoders is lower bounded by

$$R_{ck} \geq \log_{2} \left( 1 + \frac{P||\lambda_{ckk}||^{2}}{\sigma^{2}} G(\overline{U}_{c}) \right),$$

where $G(\overline{U}_{c}) = 4 \left( \frac{\sigma_{k,k}^{2}(\overline{U}_{c})}{\sigma_{k,k}^{2}(\overline{U}_{c})} + \frac{\sigma_{k,k}^{2}(\overline{U}_{c})}{\sigma_{k,k}^{2}(\overline{U}_{c})} + 2 \right)^{-1}$, with $\sigma_{k,k}(\overline{U}_{c})$ and $\sigma_{k,k}(\overline{U}_{c})$ the maximum and minimum singular values of $\overline{U}_{c}$.

Proof: Consider the system model in Section II, and the multi-layer precoding scheme in Section IV, then the received signal at user $k$ of cell $c$ is given by (14). Under Assumption 1, the approximation sign in (12) becomes equality and we get

$$y_{ck} = ||h_{ckk}||^{2} (\mathcal{Y}_{c})_{k,k} s_{ck} + n_{ck},$$

by which the achievable rate of user $k$ in cell $c$ will be

$$R_{ck} = \log_{2} \left( 1 + \frac{P||h_{ckk}||^{2}}{\sigma^{2}} [\mathcal{Y}_{c}]_{k,k}^{2} \right).$$

The value of $[\mathcal{Y}_{c}]_{k,k}$ is adjusted to satisfy the precoding power constraint $[\mathcal{F}_{c}]_{k,k}^{2} = 1$. Using a similar analysis to that in Appendix A of [2], we get $[\mathcal{Y}_{c}]_{k,k} = \sqrt{\frac{\lambda_{ckk}}{(\mathcal{U}_{c}^{T}\mathcal{U}_{c})}_{k,k}}$, where the matrix $\mathcal{U}_{c}^{T}\mathcal{U}_{c}$ can be further proved to be a positive semi-definite matrix with probability 1 using a similar proof to Lemma 2 in [2]. Hence, according to Lemma 3 in [2]

$$\mathcal{U}_{c}^{T}\mathcal{U}_{c}^{-1} \leq \frac{1}{\lambda_{ckk}} \left( \frac{\sigma_{k,k}^{2}(\overline{U}_{c})}{\sigma_{k,k}^{2}(\overline{U}_{c})} + \frac{\sigma_{k,k}^{2}(\overline{U}_{c})}{\sigma_{k,k}^{2}(\overline{U}_{c})} + 2 \right)$$

In addition to characterizing a lower bound on the achievable rates by the proposed precoding scheme, the bound in (15) separates the dependence on the channel gain $\lambda_{ckk}$, and the channel eigenspace, which is used by the following corollary to claim the asymptotic optimality of the proposed scheme.

Corollary 3: Let $R_{ck} = \log_{2} \left( 1 + \frac{P||h_{ckk}||^{2}}{\sigma^{2}} \right)$ denote the single-user rate of user $k$ in cell $c$. With the multi-layer precoding scheme in Section IV, and given Assumption 1, the achievable rate by any user $k$ in cell $c$ satisfies

$$\lim_{N_{V} \to \infty, r/N_{V} = \text{const.}} R_{ck} = \tilde{R}_{ck}$$

The proof is similar to that in Appendix B of [2]. Corollary 3 indicates that as the number of vertical antennas increases, the achievable rate of the proposed multi-layer precoding scheme approaches the single-user rate, i.e., the inter-cell and multi-user interference vanish. Further, with a large number of antennas, the decoupling assumption (Assumption 1) becomes valid with high probability thanks to the directionality of large-scale MIMO channels, as will be shown by simulations in Section VII. This illustrates how the proposed multi-layer precoding enables inter-cell interference management with low channel knowledge requirements.

VI. HYBRID ANALOG/DIGITAL IMPLEMENTATION

Thanks to the multiplicative structure and the specific design of the multi-layer precoding in Section IV, we note that each precoding layer has less dimensions compared with the prior layers. This allows the multi-layer precoding matrix to be implemented using a small number of RF chains with a hybrid analog/digital precoding architecture [2]. In this section, we brieﬂy highlight the main idea of the hybrid analog/digital implementation.

Considering the elevation multi-layer precoding matrix $\mathcal{F}_{c}^{E} = \mathcal{F}_{c}^{E(1)} \mathcal{F}_{c}^{E(2)} \mathcal{F}_{c}^{E(3)}$, we note that the dimensions of $\mathcal{F}_{c}^{E(3)}$ are $K \times K$. Hence, if $\mathcal{F}_{c}^{E(1)}$ and $\mathcal{F}_{c}^{E(2)}$ are implemented using analog hardware, and $\mathcal{F}_{c}^{E(3)}$ is implemented in the baseband, we will need only $K$ RF chains. To do that, we propose to leverage the downtilt directional antenna patterns included in the ITU channel models [6]. We assume that each antenna is an antenna port with a directional pattern and electrically adjusted downtilt angle [6], [9]. The 3GPP antenna port elevation gain $G^{E}(\theta)$ is defined as [6]

$$G^{E}(\theta) = G_{\text{max}}^{E} - \min \left\{ 12 \left( \frac{\theta - \theta_{\text{d}}}{\theta_{\text{d}}}{\text{SL}} \right)^{2} \right\},$$

where $\theta_{\text{d}}$ is the downtilt angle, and SL is the sidelobe level. Therefore, one way to approximate $\mathcal{F}_{c}^{E(1)}$ is to adjust the downtilt angle $\theta_{\text{d}}$ to force the transmission to be in the interference null-space $U_{c}^{N_{c}}$.

Once $\mathcal{F}_{c}^{E(1)}$ is implemented, the second and third layers, $\mathcal{F}_{c}^{E(2)}$, $\mathcal{F}_{c}^{E(3)}$, can be approximated exactly as proposed in [2], i.e., each one of the $K$ columns of $\mathcal{F}_{c}^{E(2)}$ can be approximated by a beamsteering vector taken from a codebook that captures the analog hardware constraints, and the $K \times K$ matrix $\mathcal{F}_{c}^{E(3)}$ is implemented in the baseband to manage the multi-user interference based on the effective channel that includes the effect of the first and second precoding layers.

VII. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed multi-layer precoding scheme. First, we consider the single-cell setup of a BS with a 2D antenna array serving $K = 4$ users in Fig. 2. The channels are assumed to have the Kronecker model in (4), with IID Rayleigh azimuth channel vectors, rank-one elevation channels, and SNR=0 dB. Fig. 2 shows the achievable rate of the proposed multi-layer precoding scheme with different numbers of BS antennas. It illustrates that the achievable rate approaches the single-user rate with
large antennas as suggested by corollary 3. Further, when a hybrid analog/digital approximation is considered with analog phase shifter quantization, the figure indicates that the number of quantization bits should scale with the number of antennas to avoid a performance degradation.

To evaluate the multi-layer precoding in a network setting, we considered a cellular setup in Fig. 3 of BS’s and MS’s that are spatially distributed according to Poisson point processes with MS’s densities 30 times the BS densities. Each MS is associated to the nearest BS, and each BS selects $K = 4$ MS’s to be served. BS’s have 2D arrays of 50m height. Channels are modeled as in (4) with IID Rayleigh azimuth channel vectors, rank-1 elevation channels, and a path-loss exponent of 3.5. Fig. 3 compares the performance of the proposed multi-layer precoding with a conventional TDD massive MIMO solution. In this conventional solution, each BS estimates the channels during the uplink training phase by correlating the received signal with the pilot sequence, which is assumed to be reused among the cells. A conjugate beamforming based on the estimated channel is then used in the downlink data transmission phase. The figure indicates that multi-layer precoding has a coverage gain over the conventional massive MIMO solution. This is mainly due to the inter-cell interference management performed by the multi-layer precoding. Note that the conventional massive MIMO solution suffers from inter-cell interference due to channel estimation errors resulted from the pilot reuse among the cells. The figure also shows that the gap between the cases when averaged interference covariance or exact interference covariance is used to build the first precoding layer decreases with large antennas thanks to the the directional characteristics of large antenna channels.

VIII. CONCLUSIONS

In this paper, we proposed a multi-layer precoding solution for full-dimensional massive MIMO systems that leverages the large-dimensional channel characteristics in the elevation direction to manage inter-cell interference using low channel requirements. Thanks to the structure of the proposed precoding matrices, they can be efficiently implemented using hybrid analog/digital architectures. The achievable rate of the proposed scheme was derived, and shown to asymptotically approach the single-user rate with a large number of antennas. In a cellular setup, simulations illustrated that a good coverage gain is offered by the proposed multi-layer precoding compared with traditional massive MIMO solutions.

REFERENCES